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A STOKES' VELOCITY PHOTOGRAPHIC METHOD FOR
MEASURING THE SIZE DISTRIBUTION OF
AEROSOLS

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16. ABSTRACT <p>Aerosol particles are allowed to settle in a vertical glass-walled vessel, and their settling velocity is determined by photographing them while the light entering the camera is being chopped at a known rate. The settling velocity (v_s) of each particle can be determined from the photographs, and by applying the Stokes' law, one can calculate the radius (r) of each particle. The Stokes' law for a sphere (specific gravity ρ_p) settling freely in a quiet medium (specific gravity ρ_m and viscosity η_m) is given by the relation,</p> $v_s = 2(\rho_p - \rho_m) g r^2 / 9 \eta_m$, where g is the acceleration due to gravity. If the volume of the photographed region of the illumination is known, one can obtain the aerosol number density and size distribution. Experiments with alumina particles of given size distributions indicate that the method works accurately for aerosols with diameters $\geq 2.0 \mu\text{m}$ [or $\geq 2(4/\rho_p)^{1/2}$ for particles with specific gravity ρ_p]. One set of a typical experiment with $3.0\text{-}\mu\text{m}$ Al_2O_3 particles is presented, which shows that the measured size distribution peaks at approximately $3.0 \mu\text{m}$. The precise error limits have not been established thus far.			
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A STOKES' VELOCITY PHOTOGRAPHIC METHOD FOR MEASURING THE SIZE DISTRIBUTION OF AEROSOLS

INTRODUCTION AND THEORY

The purpose of this research was to provide a (absolute) method for measuring aerosol size distributions that was independent of the light scattering properties, which form the basis of most commercial optical particle sizers, of the particles. Such a method could therefore be useful in calibrating and testing the performance of an optical scattering sizer that has been designed and built at Wayne State University¹ in an effort to provide standard sources of known size particles for calibrating LDV systems.²

The basis of the method is the Stokes' law [1], which relates the terminal velocity (v_s) of a sphere settling freely in a quiet medium to its radius (r) and is given by the expression

$$v_s = \frac{2}{9\eta_m} (\rho_p - \rho_m) g r^2 \quad , \quad (1)$$

where ρ_p and ρ_m represent the specific gravity of the sphere and the medium, respectively, g is the acceleration due to gravity, and η_m is the viscosity of the medium. v_s is also referred to as the Stokes' velocity. The above expression for alumina particles ($\rho_p = 3.965$) settling in air ($\rho_m = 1.22 \times 10^{-3}$ and $\eta_m = 1.818 \times 10^{-4}$ poise) reduces to

$$v_s = 4.837 \times 10^{-2} r^2 \text{ (cm/sec)} \quad , \quad (2a)$$

1. Adarsh Deepak, R. Ozarski, and J. A. Thomson: A Standard Particle Sizer-Velocimeter. NASA Technical Memorandum, March 22, 1974 (to be published).

2. This effort was supported under NASA-Marshall Space Flight Center contract No. NAS8-24810.

where r is in micrometers. For settling in water ($\rho_m = 1.0$ and $\eta_m = 0.01$ poise), equation (2a) reduces to

$$v_s = 6.463 \times 10^{-4} r^2 \text{ (cm/sec)} \quad . \quad (2b)$$

The numerical values of v_s versus r are given in Table 1 and are represented graphically in Figure 1.

Aerosol particles accelerate to this limiting (terminal) velocity rapidly. The distance and time a spherical particle falls to reach 67 percent of the terminal velocity are as follows: for alumina particles settling in air,

$$\text{Time} = \frac{v_s}{g} = 0.493 \times 10^{-4} r^2 \text{ (sec)} \quad , \quad (3a)$$

and

$$\text{Distance} = \frac{v_s^2}{2g} = 1.192 \times 10^{-6} r^4 \text{ (cm)} \quad ; \quad (3b)$$

for alumina particles settling in water,

$$\text{Time} = \frac{v_s}{g} = 0.658 \times 10^{-6} r^2 \text{ (sec)} \quad , \quad (4a)$$

and

$$\text{Distance} = \frac{v_s^2}{2g} = 2.129 \times 10^{-10} r^4 \text{ (cm)} \quad . \quad (4b)$$

Thus it takes only a few milliseconds and a fraction of a centimeter for the alumina particles ($\rho_p = 3.965$) to reach their terminal velocity.

TABLE 1. VALUES OF STOKES' VELOCITY FOR ALUMINA PARTICLES OF DIFFERENT RADII SETTLING IN AIR AND WATER.

For alumina particles,					
$\rho_p = 3.965$ (in water at 25°C).					
For air,					
$\rho_m = 1.22 \times 10^{-3}$					
and					
$\eta_m = 1.818 \times 10^{-4}$ poise (dyne-sec/cm ²).					
For water,					
$\rho_m = 1.0$					
and					
$\eta_m = 0.01$ poise.					
Stokes' velocity in air,					
$v_s = 4.554 \times 10^{-1} r^2$ (mm/sec).					
Stokes' velocity in water,					
$v_s = 6.457 \times 10^{-3} r^2$ (mm/sec).					
Particles of Substance	ρ_p	Radius, r (μm)	Diameter, 2r (μm)	Stokes' velocity, v_s	
				Air (mm/sec)	Water (mm/sec)
Al ₂ O ₃	3.965	0.15	0.3	1.02×10^{-2}	1.45×10^{-4}
		0.5	1.0	1.14×10^{-1}	1.61×10^{-3}
		1.0	2.0	4.55×10^{-1}	6.46×10^{-3}
		1.5	3.0	1.02	1.452×10^{-2}
		2.0	4.0	1.82	2.583×10^{-2}
		2.5	5.0	2.84	4.03×10^{-2}
		3.0	6.0	4.098	5.811×10^{-2}
		4.0	8.0	7.286	1.033×10^{-1}
		5.0	10.0	11.38	1.61×10^{-1}

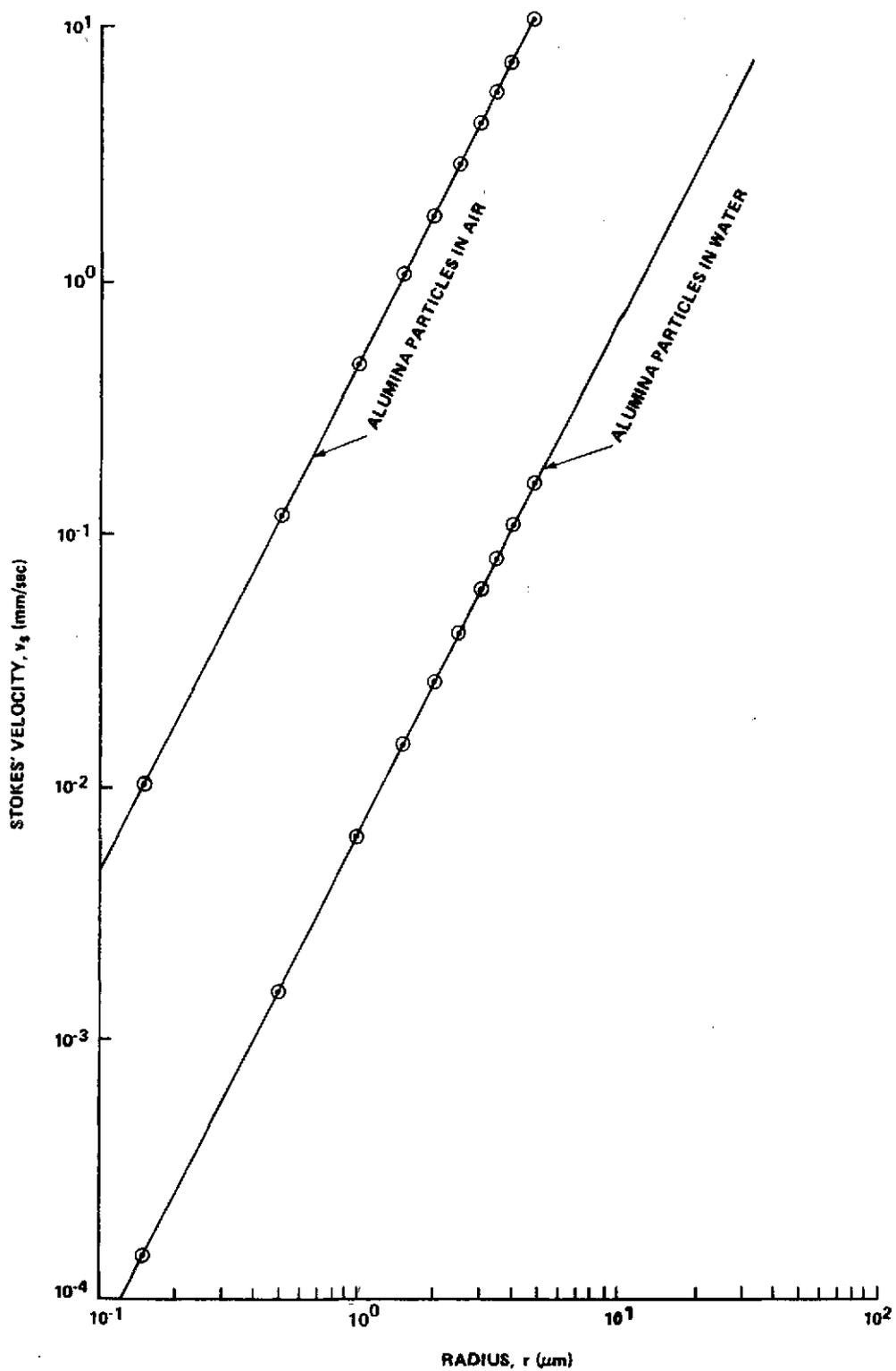


Figure 1. Stokes' velocity versus radius.

GENERAL DESCRIPTION OF THE METHOD

Aerosol particles are allowed to fall freely in a vertical glass-walled cell (Fig. 2) in which the convection currents have been reduced to a minimum. The settling velocity of the particles is determined by photographing them while the light entering the lens is chopped at a known rate. The particles are illuminated by a 1- or 2-mm thick vertical sheet or slab of light projected into the cell. A camera aimed perpendicular to the slab of light photographs the particle tracks; typical exposure times are 0.5 or 1 sec. The image of the falling particle is thus a series of dashes (with the chopper) (Fig. 3). The velocity of fall can then be measured from the spacing of the dashes.

Under ordinary conditions the velocity thus measured is the sum of the Stokes' velocity and the convection velocity. It is the experimental suppression of the convection currents and the theoretical elimination of their effects that is the major cause of the complexity of the experiment and of the limit of the applicability of the method to large particles. The procedures for damping the convection currents are described later. One finds it very difficult to reduce the convection velocity below about 0.5 mm/sec. However, it is possible to eliminate by subtraction most of the effects of convection by ensuring the presence of many submicron particles (such as DOP particles approximately 0.3 μm in diameter). These move essentially with the convection air currents and can be used to measure the vertical component of convection velocities, which can then be subtracted from the vertical component of the total velocity of large particles to obtain their true Stokes' velocity.

BRIEF DESCRIPTION OF THE INSTRUMENTATION

The instrument setup essentially consisted of a vertical glass-walled cell and a photographic system (Fig. 4).

The cell was a vertical glass-walled chamber which had airtight joints and can be closed-off airtight at the top and bottom. Its cross section can be octagonal, rectangular, or square. The one used for this research was octagonal (2-in. sides) in cross section and 18 in. high. Its walls were covered on the inside with (black) flock paper except for the appropriate 2-in. diameter openings for the passage of light and for the camera lens. This helped minimize the extraneous light entering the lens and provided a dark backdrop for the illuminated particles. The bottom was closed off by inserting a 1-in. thick aluminum plate.

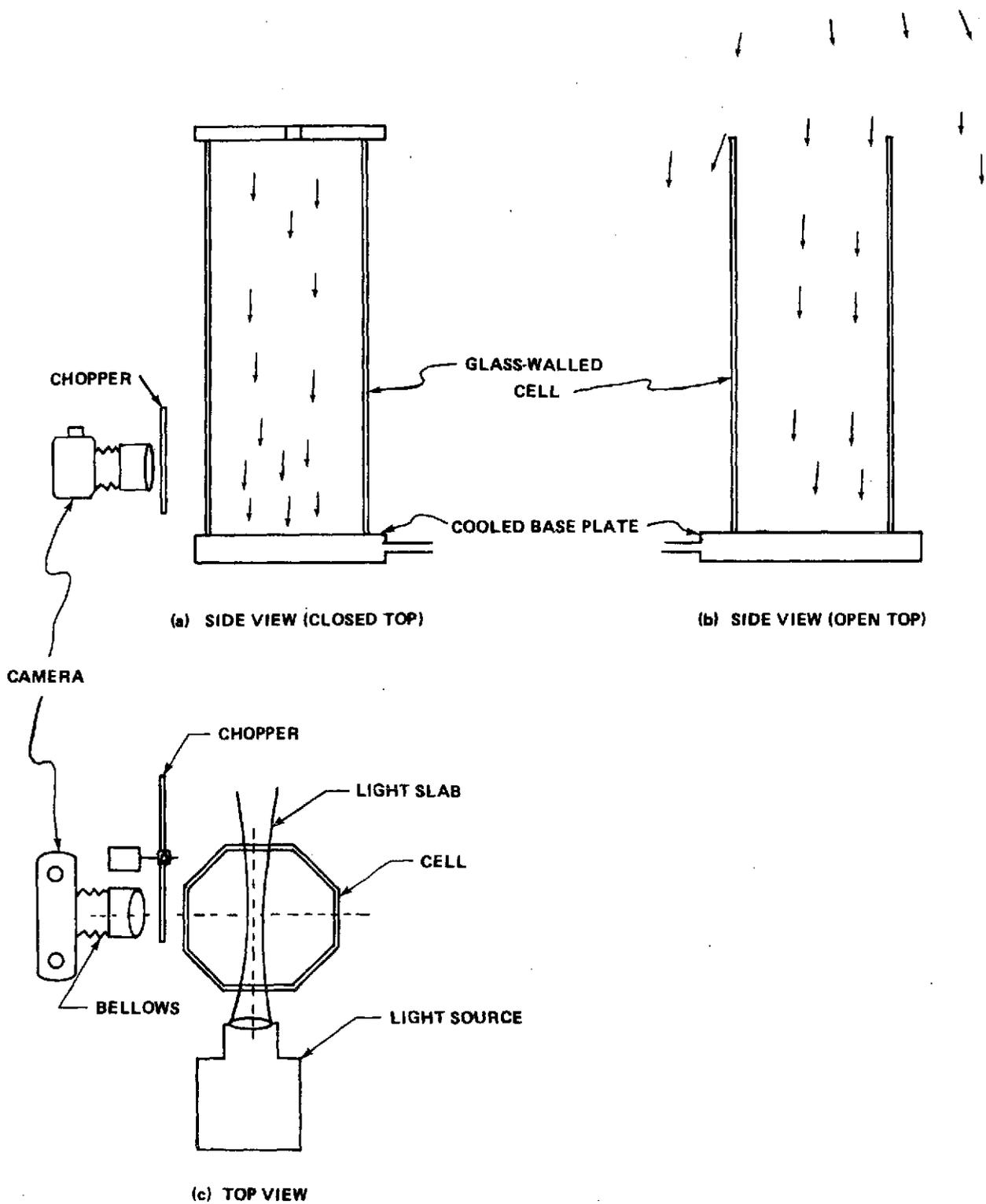


Figure 2. Schematic illustration of the method.

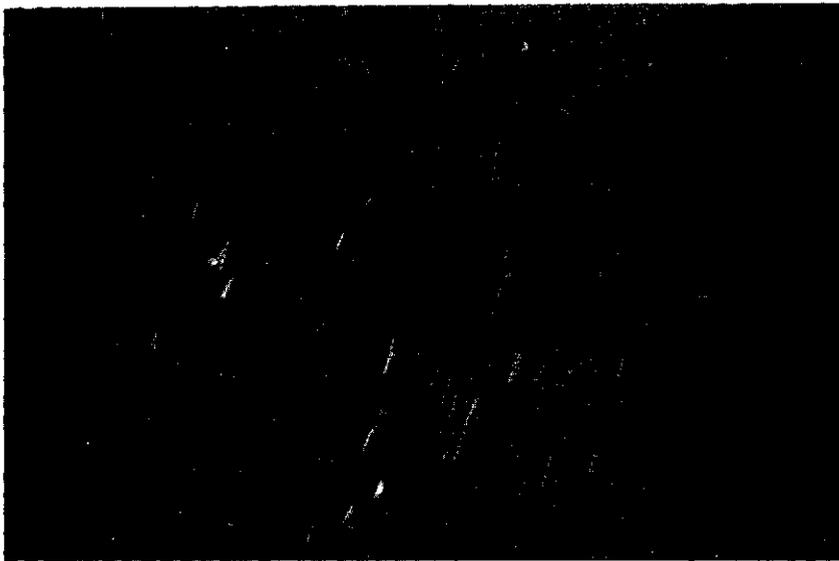


Figure 3a. A typical photograph of settling Al_2O_3 particles.

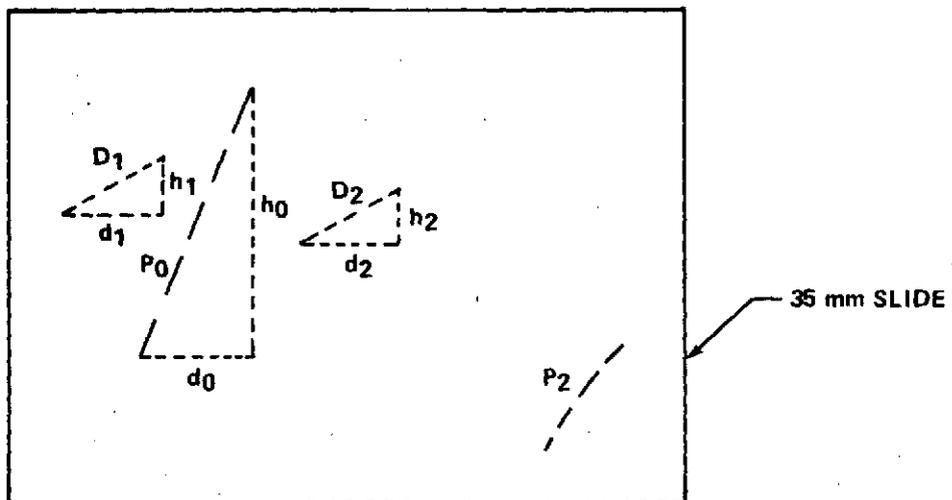


Figure 3b. Typical tracks for 0.5-sec exposure (schematic).

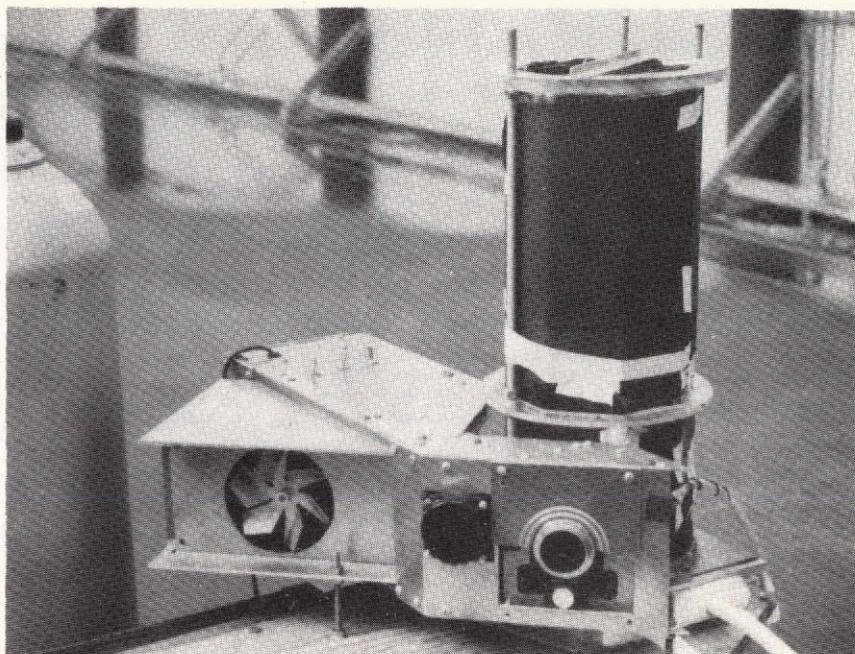


Figure 4a. Photograph of the experimental setup.

The plate can be cooled to and maintained at a steady temperature, the temperature of dry ice (-12°C), over its entire surface by circulating a CO_2 and acetone mixture through a passage inside the plate using a dry ice-and-acetone pump (Fig. 5).

The photographic system consisted of a light source, a camera with a bellows, and a uniformly rotating chopper. The entire system was mounted between two parallel aluminum sheets (0.125 in. thick) held 5 in. apart by aluminum rods to form a sturdy unit for field use.

The source of light was a 500-W slide projector (Bell and Howell) with a rectangular aperture (1 mm by 1.5 in.) inserted in the position of a slide. The beam of light was focused by a short-focused convex lens ($f/2$) to form a sharp focus of the aperture at the center of the cell. Near the focus a vertical sheet or slab of light 1 or 2 mm thick illuminated the falling particles. The depth of focus was about 5 mm.

The camera unit was an $f/1.7$ Minolta lens mounted in front of a bellows, and the unit was fixed to the external framework (Fig. 4). The bellows was adjusted to obtain a sharp image of an illuminated object at the position of the

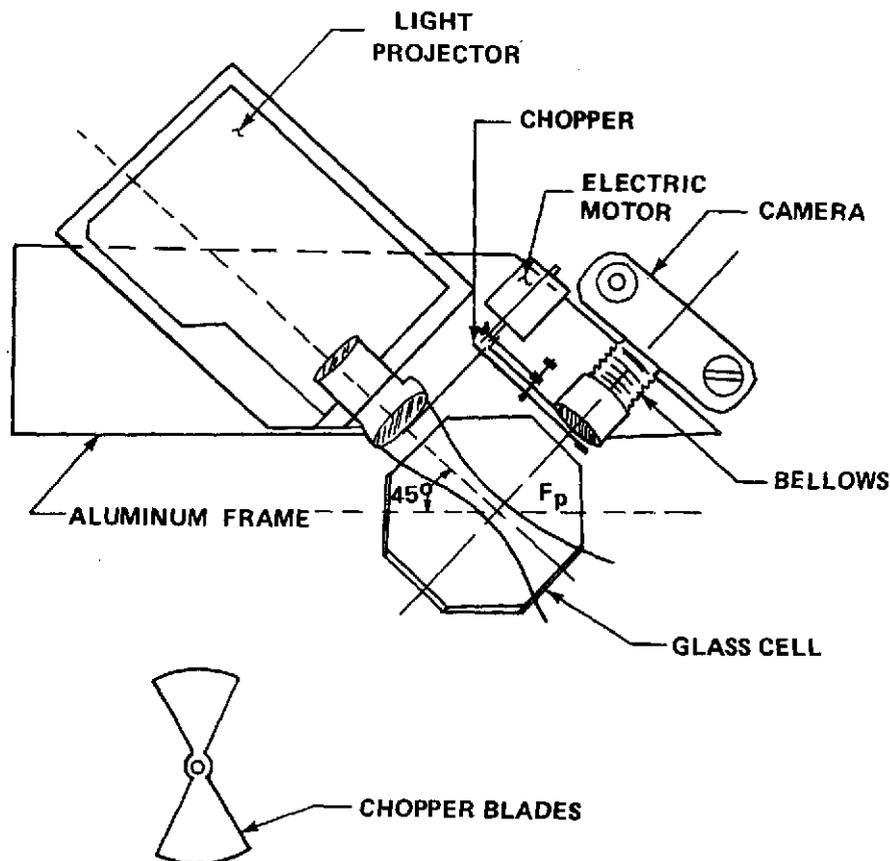


Figure 4b. Schematic diagram of the instrumentation.

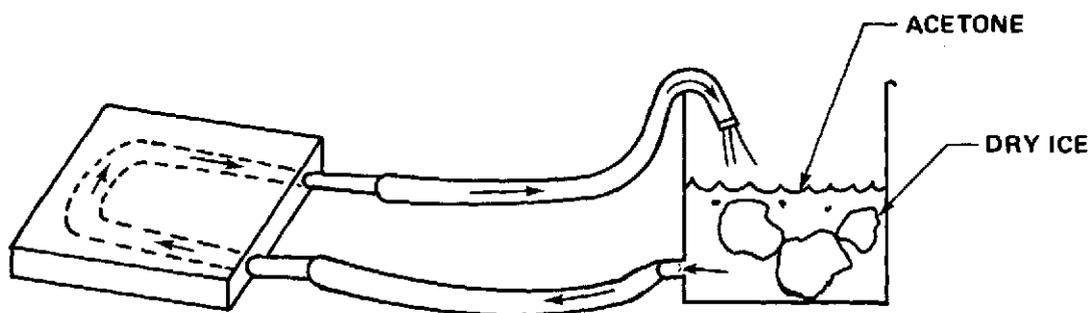


Figure 5. Acetone and dry ice pump.

projector beam focus, approximately 3.5 in. in front of the lens, so that maximum magnification could be obtained. The magnification in this case was approximately 0.8. The camera body could be removed from the bellows unit for film changing.

A uniformly rotating chopper was used in front of the lens to accurately select and measure the particle track lengths in the photograph. With the aid of a belt-and-pulley arrangement the chopper (2 or 4 blades) was rotated by a small synchronous electric motor (300 rpm) fixed to the external framework. By choosing an appropriate combination of pulleys and chopper blades, one could obtain different chopping rates (typical values were 10 or 100 chops/sec).

Tri-X (ASA 400) film was found to be the most suitable film considering speed, resolving power, and the ease of developing. It is sensitive (fast) enough to record tracks of particles of diameters as low as $0.3 \mu\text{m}$ often moving with velocities as high as 3 mm/sec, when the bellows was set for a magnification of approximately 0.8.

A SETTLING EXPERIMENT WITH ALUMINA PARTICLES

An extensive series of experiments using alumina particles was carried out to test the performance of the method and to define the limits of its accuracy and applicability. The samples of Al_2O_3 particles used were not strictly monodisperse samples, but had size distributions which were narrowly polydispersed about the mode diameter values of $0.3 \mu\text{m}$, $1.0 \mu\text{m}$, and $3.0 \mu\text{m}$.³ These samples were chosen over the more accurately calibrated particles (such as latex particles) because they were relatively inexpensive and provided enough calibration for the purposes of making the initial feasibility and performance studies of the method. Only one set of results of a typical experiment with $3.0\text{-}\mu\text{m}$ particles is presented in this report.

Aerosols of Al_2O_3 were generated by impinging a jet of air on a small sample of Al_2O_3 particles placed inside a long vertical tube (3-in. diameter, 5-ft length). The Al_2O_3 aerosols were drawn off into the glass cell as needed.

After the particles were introduced into the cell, the top was closed airtight. After turning the projector light on, several bright specks of light could be seen through the camera viewfinder swirling around as they traversed through the beam. These random flow patterns were created to some extent by the initial disturbance created while introducing the particles in the cell, and to a large extent by the convection air currents. These convection currents

3. The data were provided by the supplier, Adolf Meller Co., Rhode Island.

arise because of the small differences in the temperatures of the various parts of the cell. As mentioned before, the velocity of the particle is then the sum of the Stokes' velocity and the convection velocity.

It is essential to the effective working of the method to suppress convection air currents. One resorts to several procedures to reduce the currents. The simplest procedure is to make all the joints of the cell truly airtight. The best test for air leakage is to check for water leakage. Also, closing the top of the cell helps.

The most effective procedure for damping the convection currents is to cool the bottom of the cell with ice, dry ice, or dry ice and acetone. This creates a stabilizing temperature gradient, and consequently density stratification, in the column of air in the cell. The best results were obtained by using the acetone-dry ice pump to cool the plate to -12°C . The motion of the falling particles was seen to quickly stabilize to a nearly vertical direction of fall within 90 sec after the insertion of particles.

In spite of taking all these precautions, slow motion convection currents (~ 1 mm/sec) still seemed to persist. One can measure their velocities by introducing $0.3\text{-}\mu\text{m}$ diameter DOP particles (Royco DOP generator) into the cell. These particles essentially follow the convection currents, since their Stokes' velocities (~ 0.005 mm/sec) are negligibly small compared to the convection velocities (typically 1 mm/sec). The same is true with $1.0\text{-}\mu\text{m}$ or $0.3\text{-}\mu\text{m}$ diameter Al_2O_3 particles. It is difficult to distinguish their tracks from those of DOP particles, making the measurements unreliable. The DOP particles have been seen to follow the currents in the airtight cell for periods of over 8 hr. The tracks of $3.0\text{-}\mu\text{m}$ diameter Al_2O_3 particles, however, are easily distinguishable from those of DOP aerosols. It appears that had we been able to use $2.0\text{-}\mu\text{m}$ diameter Al_2O_3 particles, then, with some care, their tracks could also be distinguishable from DOP tracks. Thus it appears that the lower limit of diameters for which this method is accurate is $3.0\ \mu\text{m}$, but the limit can be, perhaps, decreased to $2.0\ \mu\text{m}$ for particles whose specific gravity is approximately 4.0 (or $\sim 2\left(\frac{4}{\rho_p}\right)^{1/2}$, for specific gravity ρ_p).

When the particles stabilized to a nearly vertical direction of fall, photographs were taken for 0.5-sec or 1-sec exposure times. Several photographs for each sample were necessary to ensure statistical accuracy of the results.

It is important to periodically check during the experiment and make certain portions of the glass and the camera lens do not become fogged.

THE PHOTOGRAPHIC DATA REDUCTION

The negatives were projected upon a screen and the lengths of the selected particle tracks were measured by hand. The visual distinction between the DOP tracks and the Al_2O_3 tracks was made by noting that all of the DOP tracks in a given region of the frame were of more or less uniform lengths and had the same inclination to the vertical, while the tracks of the heavier Al_2O_3 particles were larger and less inclined to the vertical. Notice that the horizontal components for all the tracks in the given region should be the same length. An exception to this would be a very large particle (typical diameters $> 20.0 \mu\text{m}$) which might travel almost vertically through the field of view.

The selection of the Al_2O_3 tracks suitable for measurements was made as follows. (A few tracks that appear typical are schematically drawn in Figure 3b.) For a chopper speed of 10 chops/sec, one expects, for a 0.5-sec exposure, a series of five dashes to represent the track of each particle if the particle falls vertically and remains within the 1-mm thick illumination slab during the 0.5 sec. If however, the particle trajectory is transversely inclined to this slab of light such that the particle remains illuminated for only a fraction of the exposure time, then it may leave a track such as P_2 (Fig. 3b) having a number of dashes less than five. If the number of dashes is less than three (for 0.5-sec exposure) the particles are excluded from measurement. However, it was found that under the best working conditions, the fraction of such particles is rather small and their exclusion introduces a small error in the particle number density results. Thus the chopping of light not only helps one make an accurate measurement of particle velocities but also aids in the selection of particles that remain illuminated throughout the exposure.

Ideally, the convection velocity component should be measured in the vicinity of each of the different Al_2O_3 tracks by selecting two DOP tracks preferably close to and on opposite sides of an Al_2O_3 track. This is illustrated in Figure 3b by the DOP tracks, D_1 and D_2 , about the Al_2O_3 track, P_0 . The lengths of the horizontal components (d_1 , d_2 , d_0) of the three tracks should be equal. Then the true Stokes' velocity component is

$$h_s = h_0 - \left(\frac{h_1 + h_2}{2} \right) ,$$

where h_0 , h_1 , and h_2 are vertical components of their track lengths. The Stokes' velocity

$$v_s = h_s/t \quad ,$$

where t is the exposure time. Using the Stokes' law, one obtains the radius, r .

DATA ANALYSIS

To analyze the data, the particle tracks in a number of photographs are measured and reduced to particle velocity (and consequently particle radius). To obtain a 10-percent error bar in each of the 10, for example, particle radius intervals, it is necessary to reduce about 1000 tracks (± 20 percent in 10 intervals requires about 250 tracks).

If the particle tracks are all entirely contained within the viewed volume, the average particle size distribution can be evaluated as shown below.

The height of the viewed region = h .

The area of the horizontal cross section viewed region = A .

The volume of the viewed region = hA .

If $N_p(r_i, \Delta r)$ is the number of particles seen in the p th photograph for which the particle radius lies between $r_i - \frac{\Delta r}{2}$ and $r_i + \frac{\Delta r}{2}$ and N_f is the total number of frames viewed, then the size distribution for the i th radius interval is given by

$$[n(r)]_i = \frac{1}{N_f} \sum_p \left[\frac{N_p(r_i, \Delta r)}{hA \Delta r} \right]$$

$$[n(r)]_i = \frac{1}{N_f h A \Delta r} \sum_p N_p (r_i, \Delta r) \quad ,$$

if h , A , and Δr are constant.

For particles whose tracks do not both originate and/or terminate in the illuminated volume, the procedure is to count the number of tracks, $N_p (r_i, \Delta r; z_j)$, which pass through a given elevation z_j in the cell during the exposure time. If measurements are made at the number (N_z) of levels in the photograph, then the particle size distribution in this case is given by⁴

$$[n(r)]_i = \frac{1}{N_f N_z} \sum_p \frac{N_p (r_i, \Delta r; z_j)}{[v_s (r_i) \Delta t_p] A \Delta r} \quad ,$$

where $v_s (r_i)$ is the velocity and Δt_p is the exposure time. $N_p (r_i, \Delta r; z_j)$ is the number of particles crossing lines at each of the z_j levels, regardless of whether the same particle has been counted on other levels.

DISCUSSION OF RESULTS

The results of experiments with only the 3.0- μm diameter Al_2O_3 particles are presented here. The measurements of track lengths were done by hand from the negative which was enlarged by projecting on a screen. The number density of the particles in each of the photographs analyzed is about 175 cm^{-3} . The size distribution is given in Table 2 and Figure 6. The peak of the $n(r)$ versus r curve is at about 1.45 μm . This agrees well with the value of 1.5 μm supplied by the manufacturer for the sample of alumina particles, which are supposed to be narrowly polydispersed about 1.5 μm .

The results of experiments with Al_2O_3 particles of diameters 0.3 μm and 1.0 μm were inconclusive because it was found difficult to distinguish the Al_2O_3 tracks from the DOP tracks in the photograph. However it is felt that

4. Personal communication with Dr. J. A. Thomson, Physical Dynamics, Inc., Berkeley, California.

TABLE 2. SIZE DISTRIBUTION OF SETTLING ALUMINA PARTICLES

<p>Volume of the illuminated particles is 0.04 cm^{-3}.</p> <p>Average number density of the Al_2O_3 particles is 175 cm^{-3}.</p> <p>Average number density of the DOP particles is 5000 cm^{-3}.</p>	
Radius Range (μ)	$n(r), \text{ cm}^{-3} \mu\text{m}^{-1}$
0 - 0.5	0
0.5 - 0.75	14.3
0.75 - 1.0	28.6
1.0 - 1.25	155.0
1.25 - 1.5	186.0
1.5 - 1.75	72.1
1.75 - 2.0	85.0
2.0 - 2.5	21.8
2.5 - 3.25	9.28
3.25 - 3.75	9.46
3.75 - 4.25	4.00

for $1.0 \mu\text{m}$ particles, it would be possible to get meaningful reliability by using more sophisticated optical track measurement equipment.

Even though one can photograph $0.3\text{-}\mu\text{m}$ diameter DOP particles moving with speeds of approximately 1.0 mm/sec , it is thus the speed of the convection currents that defines the lower limit of the diameters of aerosols for which this method is accurate. For particles of alumina, it seems possible to push the lower limit of diameters to $2.0 \mu\text{m}$, and for particles of specific gravity ρ_p to $2(\sqrt{4/\rho_p}) \mu\text{m}$, without loss of accuracy.

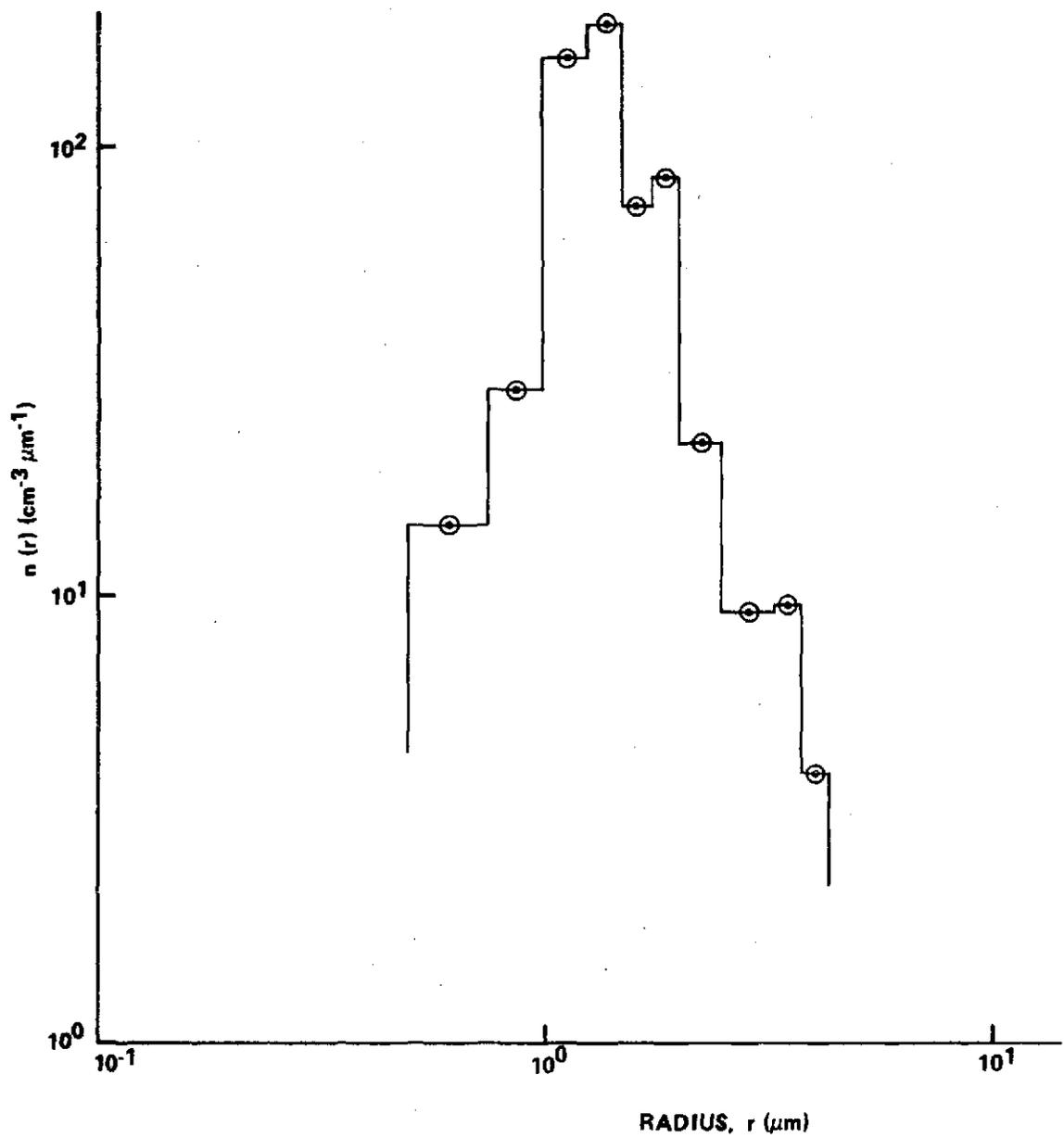


Figure 6. Graph of $n(r)$ versus r for a sample of Al_2O_3 particles of (supplier quoted) $1.5\text{-}\mu\text{m}$ radii.

REFERENCE

1. Stokes, G.G.: Trans. Cambridge Phil. Soc., vol. 9, no. 8, 1851.

APPROVAL

A STOKES' VELOCITY PHOTOGRAPHIC METHOD FOR MEASURING THE SIZE DISTRIBUTION OF AEROSOLS

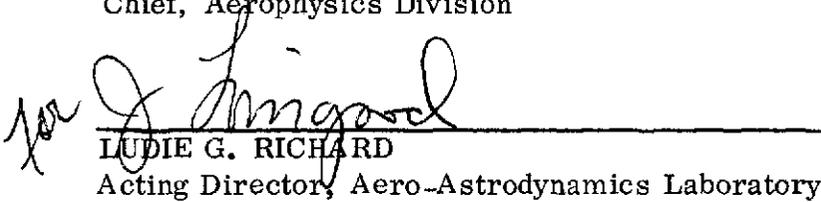
By Adarsh Deepak

The information in this report has been reviewed for security classification. Review of any information concerning Department of Defense or Atomic Energy Commission programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

This document has also been reviewed and approved for technical accuracy.



W. K. DAHM
Chief, Aerophysics Division



LUDIE G. RICHARD
Acting Director, Aero-Astrodynamic Laboratory